Helical magnetic fields from inflation and their evolution

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reference:

R Durrer, LH, RK Jain, JCAP 03 (2011) 037

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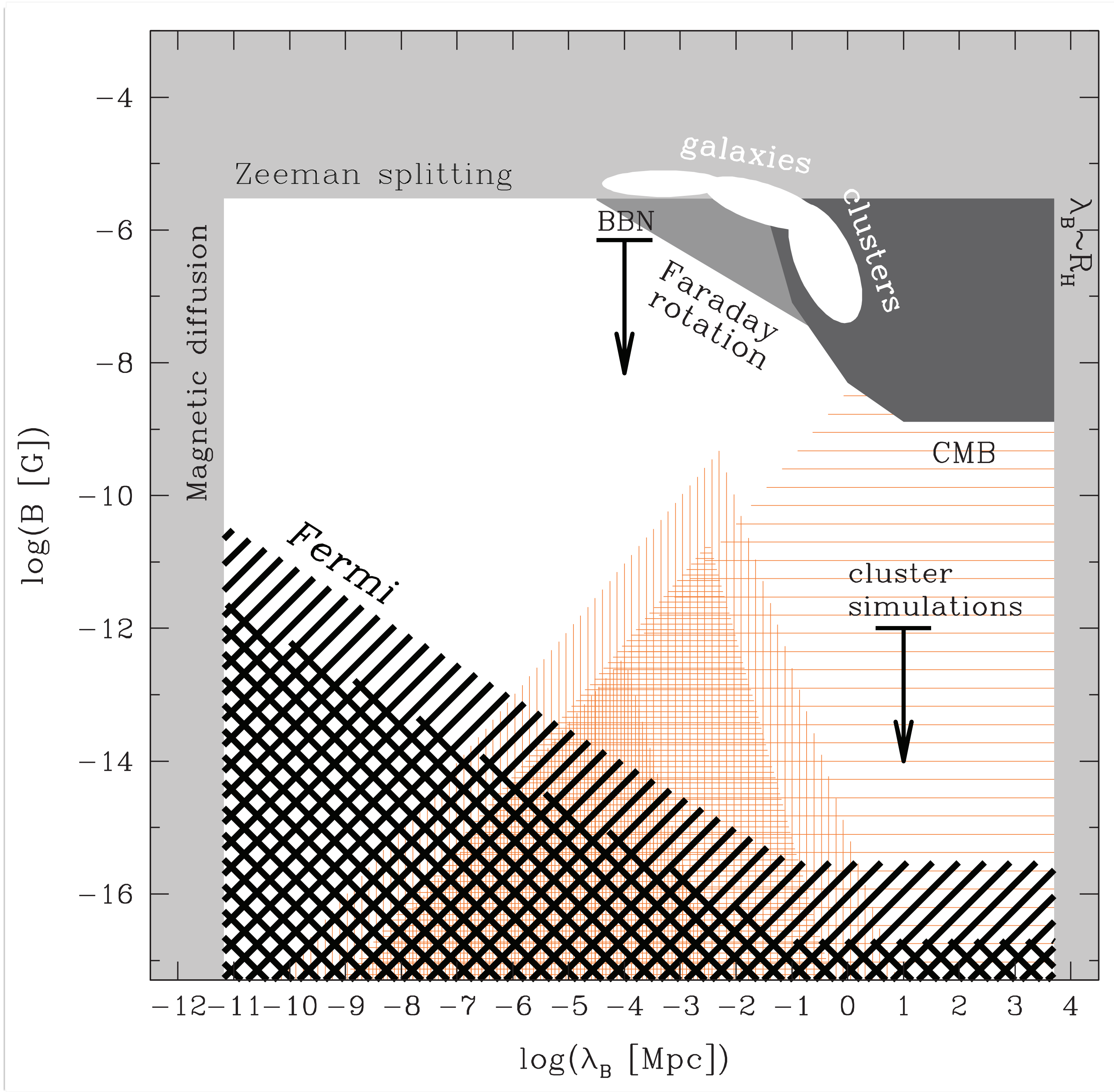
Outline

- Motivation for primordial magnetic fields
- Helical magnetic field generation during inflation
- Slow roll and beyond
- Further evolution: inverse cascade
- Conclusions

Motivation for primordial magnetic fields

Observations



from: **Neronov & Vovk**, Science 328 (2010) 73

- Magnetic fields of ~μGauss observed in galaxies, clusters & regions around high-z quasars
- CMB bounds: $B < 10^{-9}$ G
- Constrained cluster simulations: $B < 10^{-10}$ G
- Missing secondary GeV gamma-rays from TeV blazars: $B > 10^{-15}$ G

Why (helical) primordial magnetic fields?

- Primordial magnetic fields (from inflation)

  ... can produce the observed weak fields on very large scales, even in the intergalactic medium.

  ... would explain why there are magnetic fields everywhere.

- Helicity is a conserved quantity and leads to

  ... an inverse cascade in the turbulent plasma era, that can move power from small to large scales.

  ... distinct signatures in CMB, such as TE- and EB-cross-correlations.

- In many inflation scenarios helical magnetic fields are generated naturally (see Neil Barnaby and Lorenzo Sorbo's talks)

Helical magnetic field generation during inflation

Electromagnetism & inflation

- Standard EM is conformally invariant ⟶ fluctuations do not grow in conformally flat space-times such as Friedmann-Lemaître.

- Modifications could be introduced to break conformal invariance:

$$\mathcal{L} \supset -\frac{1}{4} I^2(\phi, R) F_{\mu\nu} F^{\mu\nu} + M^2(\phi, R) A_\mu A^\mu + \frac{1}{4} f(\phi, R) F_{\mu\nu} \widetilde{F}^{\mu\nu}$$

  where I(...), M(...), f(...) are possible couplings to (pseudo-)scalar fields (inflaton or not) or curvature invariants.

- As a consequence the evolution of the Fourier modes of quantum fluctuations in the photon field is modified.

Electromagnetism & inflation

- For an axial coupling to scalar inflaton:

$$\nabla_\alpha F^{\mu\alpha} = f'(\phi)\,(\partial_\alpha \phi)\,\widetilde{F}^{\mu\alpha}$$

"scalar current"

$$\nabla^\alpha \partial_\alpha \phi - V'(\phi) = \frac{1}{4} f'(\phi)\, F_{\alpha\beta} \widetilde{F}^{\alpha\beta}$$

feedback

Electromagnetism & inflation

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feedback

- Some notation:

$$\langle \widetilde{B}_i(t,\boldsymbol{k})\widetilde{B}^*_j(t,\boldsymbol{q})\rangle = \frac{(2\pi)^3}{2}\delta(\boldsymbol{k}-\boldsymbol{q})\Big\{(\delta_{ij}-\hat{k}_i\hat{k}_j)P_S(t,k) - i\epsilon_{ijn}\hat{k}_n P_A(t,k)\Big\}$$

$$\frac{d\widetilde{\rho}_B}{d\ln k}(t,k) = \frac{k^3}{(2\pi)^2}P_S(t,k)$$

magnetic energy density

$$\frac{d\widetilde{\mathfrak{H}}}{d\ln k}(t,k) = \frac{k^2}{2\pi^2}P_A(t,k)$$

helicity density

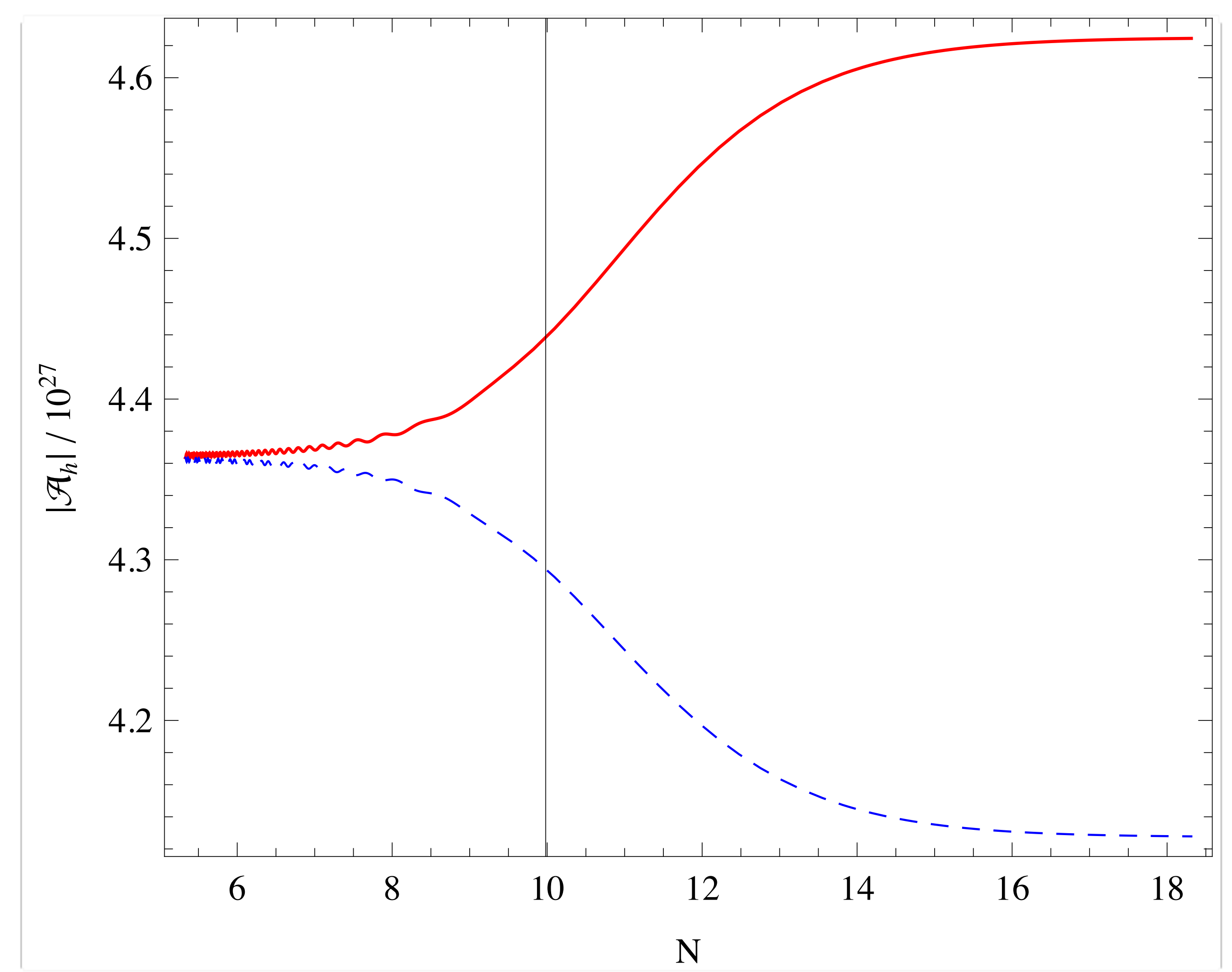
Helical vs. non-helical

Non-helical case: $\ddot{\bar{\mathcal{A}}}_\pm + (k^2 - \ddot{I}/I)\bar{\mathcal{A}}_\pm = 0\,, \quad \bar{\mathcal{A}}_\pm = I\mathcal{A}_\pm$

Helical case: $\ddot{\mathcal{A}}_\pm + (k^2 \pm k\dot{f})\mathcal{A}_\pm = 0$

- The two helicity modes evolve differently.
- The coupling term is scale-dependent & only active around horizon-crossing. On large scales:

$$\left[1 + \frac{k^2}{\mathcal{H}^2} \pm \frac{k\dot{f}}{\mathcal{H}^2}\right]\mathcal{A}_\pm \simeq 0$$



Slow roll and beyond

Slow roll

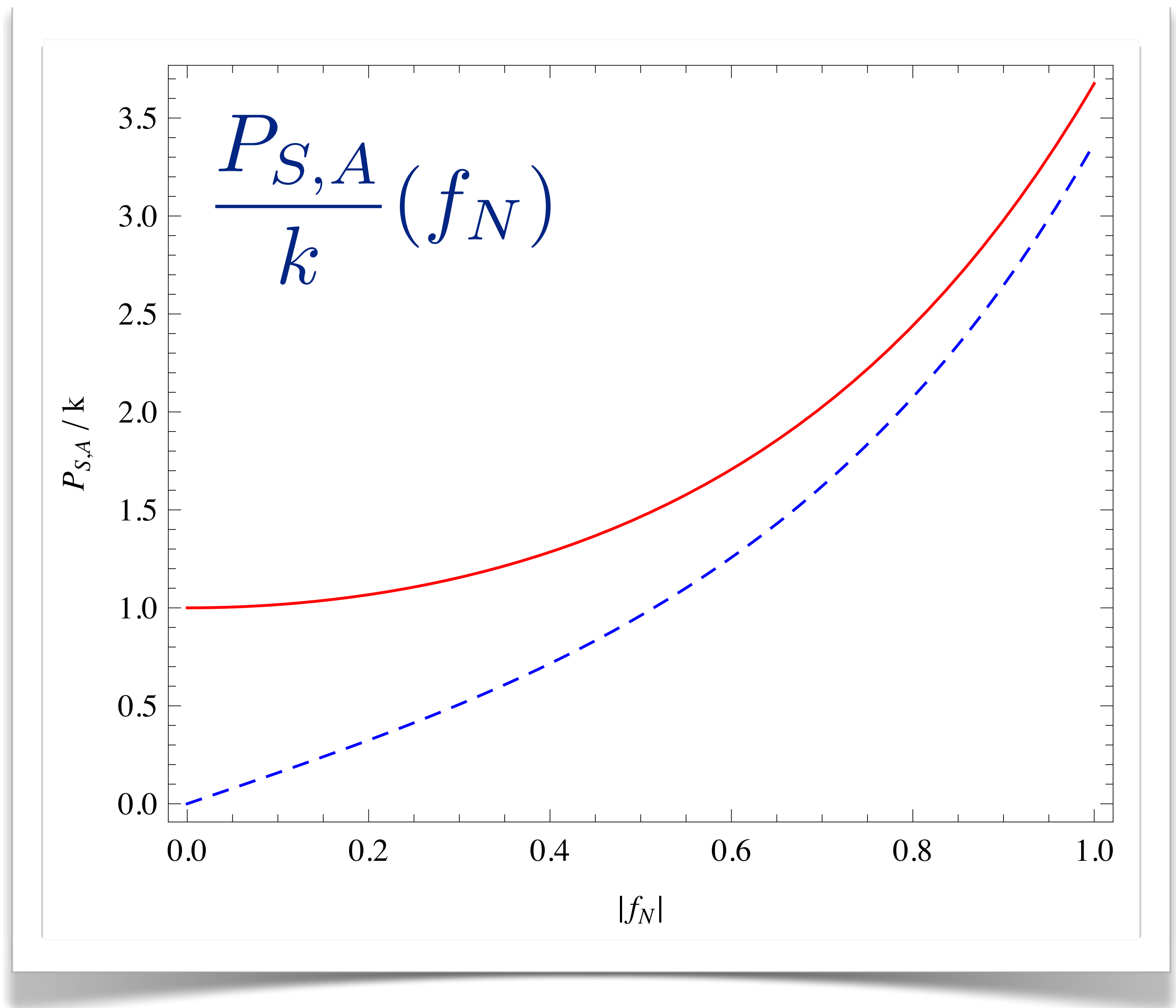
- Slowly rolling inflaton: $\epsilon \equiv \frac{m_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \quad \Rightarrow \quad \dot{\varphi} \simeq \sqrt{2\epsilon} m_p \mathcal{H}$

$$\mathcal{H} \simeq -1/t \qquad \varphi \simeq \varphi_{in} + \sqrt{2\bar{\epsilon}} m_P \ln(t/t_{in})$$

- For **constant** $f_N \equiv \dot{f}/\mathcal{H}$ the evolution of the modes can be solved analytically:

$$P_S(k) = k \frac{\sinh(\pi f_N)}{\pi f_N}$$

$$P_A(k) = k \frac{\cosh(\pi f_N) - 1}{\pi f_N}$$



Conditions on the coupling function $f(\varphi)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}$

- Is constant f_N a good approximation? (reminder: $f_N \equiv \dot{f}/\mathcal{H} = 2\xi_{\rm Sorbo}$)
  - The coupling is only active around horizon crossing (a few e-foldings):
    $\longrightarrow$ f_N can be taken constant for each mode, at least.
  - The inflaton only varies slowly during inflation: $f_N \simeq f'(\varphi)\sqrt{2\epsilon}\,m_P$
    if $f(\phi)$ is not an extremely rapidly varying function (double exponential),
    then f_N constant throughout is a good approximation.
- Naïve bound from pertubativity of the modified electromagnetism:

$$\left|\frac{S_I[\phi, A_\mu]}{S_{\rm em}[A_\mu]}\right| < 1 \qquad \longrightarrow \qquad |f_N| < 1$$

Backreaction & other bounds

- Bound from backreaction: demanding the generated electromagnetic fields not to affect the inflaton background evolution, yields

$$\mathcal{S}(f_N) \lesssim \frac{m_P}{H_{end}} \simeq \frac{m_P^2}{T_\star^2} \simeq 1.5 \times 10^{10} \left(\frac{10^{14}\,\mathrm{GeV}}{T_\star} \right)^2$$

reheating temperature

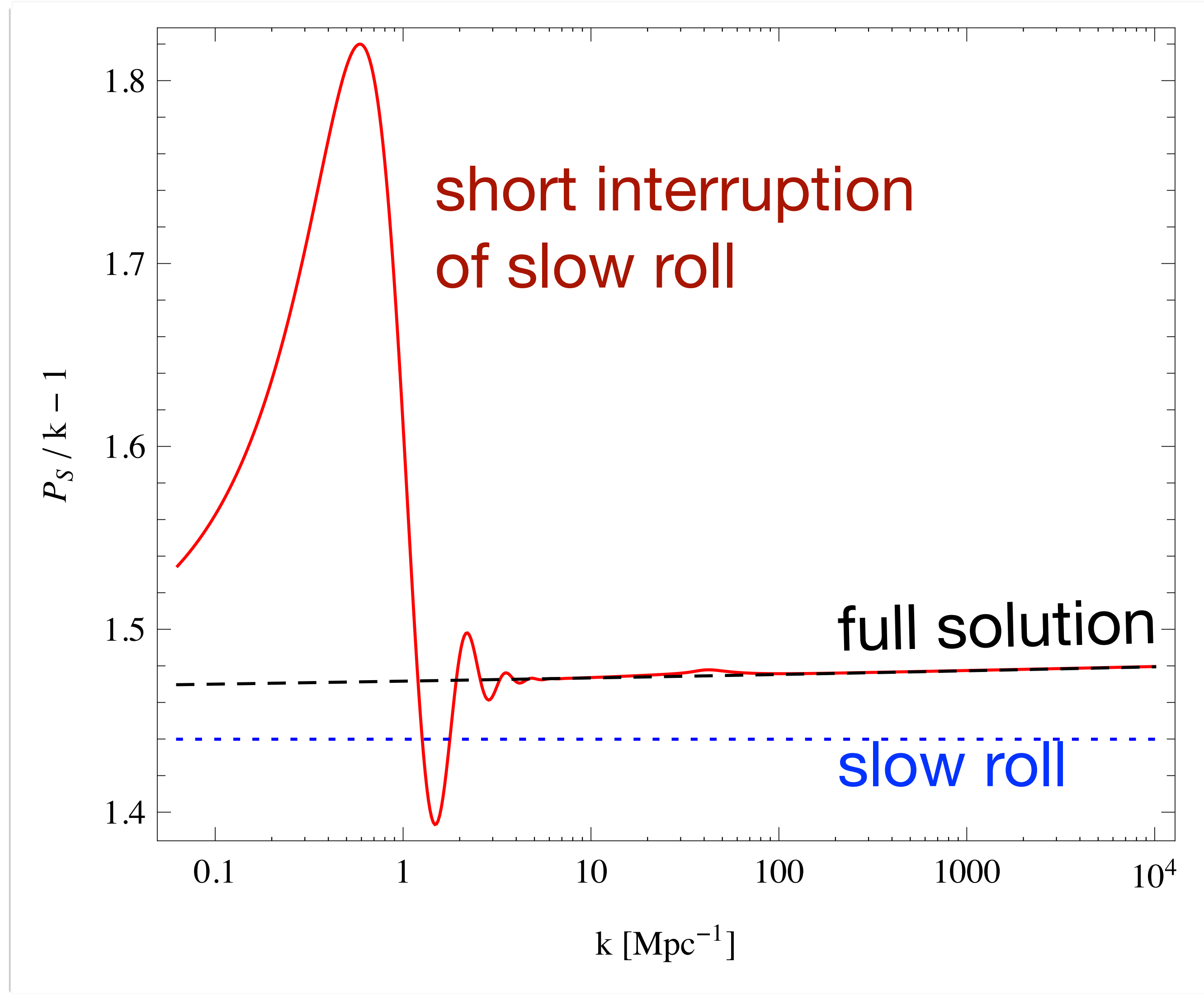
where $\mathcal{S}(f_N) \equiv \sqrt{\frac{\sinh(\pi f_N)}{4\pi^3 f_N}}$

$$\rightarrow \quad f_N \lesssim 17\, \frac{10^{14}\,\mathrm{GeV}}{T_\star}$$

- For $|f_N| \gtrsim \mathcal{O}(1)$ feedback on the evolution of inflaton perturbations leads to detectable non-Gaussianity in the primordial curvature perturbation or to a "large" scalar-to-tensor ratio. See Barnaby & Peloso 2010, Barnaby et al 2011 and Sorbo 2011 for helical GW from this scenario. Bound: $\xi \lesssim 2.5 \;\Rightarrow\; |f_N| < 5$

Deviations from slow roll

- Numerical comparison of the slow roll result with real model
- A short interruption of slow roll inflation is allowed by the CMB data, though already strongly constrained.



$$V_\beta(\varphi) = \frac{1}{2}m^2\varphi^2\left[1 + \beta \tanh\left(\frac{\varphi - \varphi_0}{\Delta\varphi}\right)\right]$$

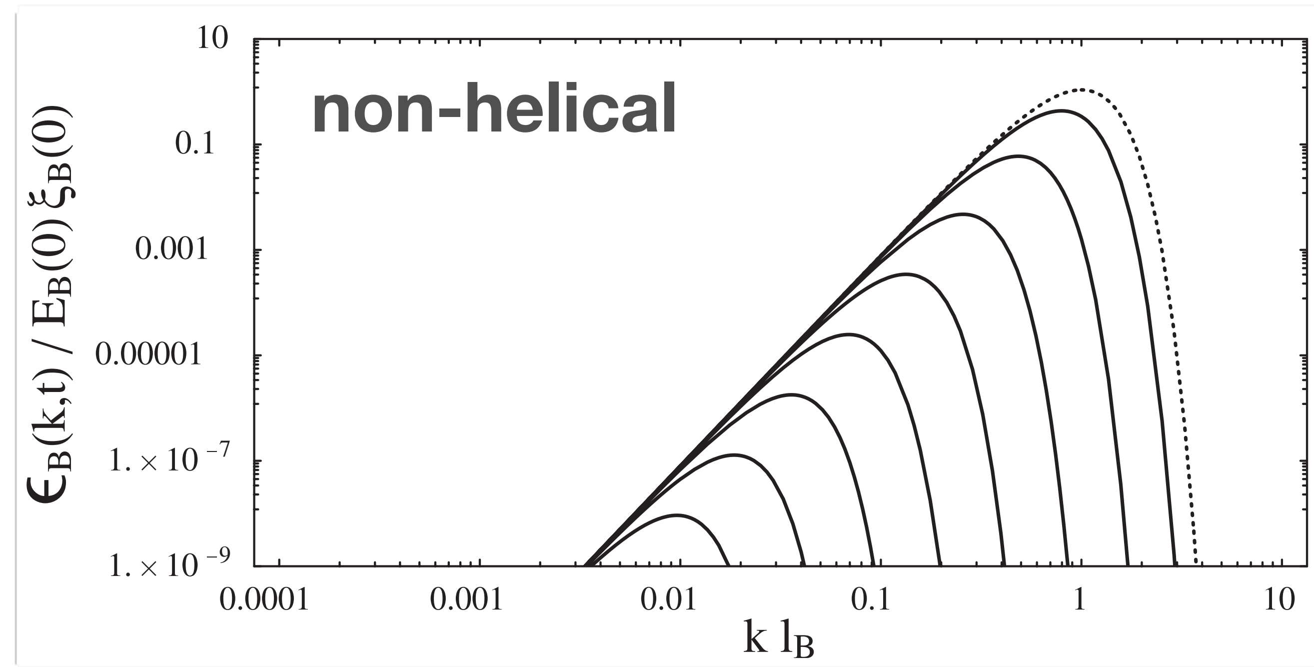
$$f(\phi) \propto \phi^2$$

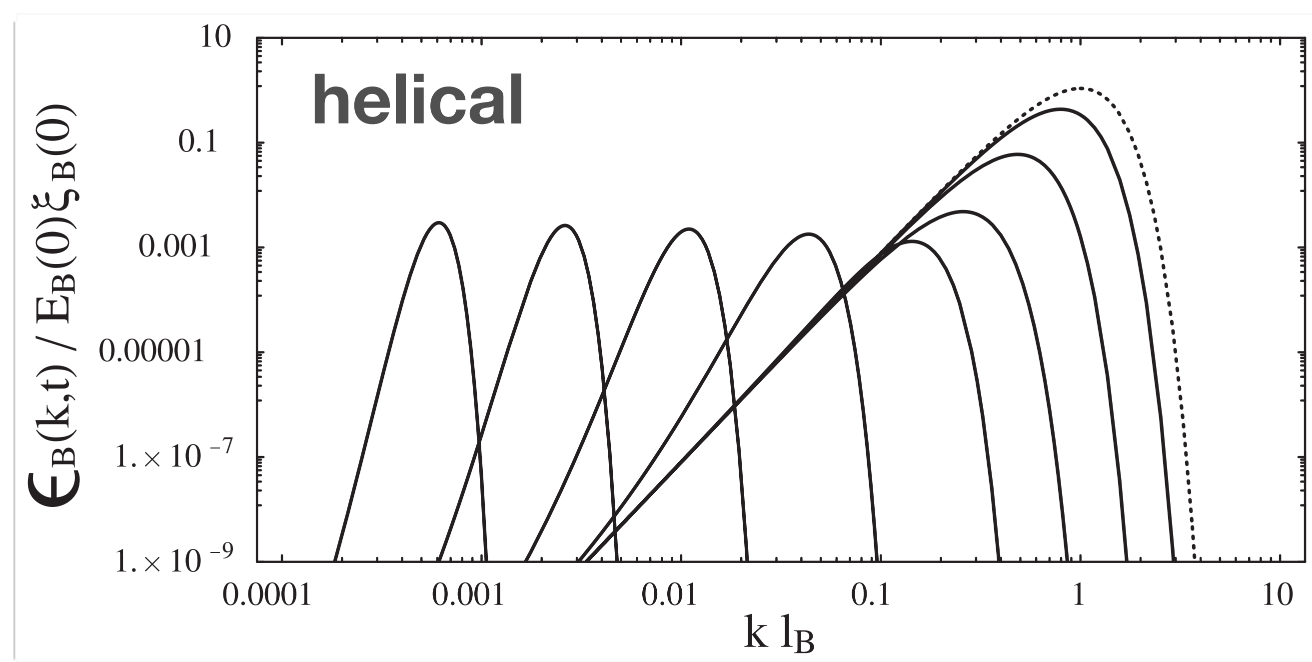
Further evolution: inverse cascade

Inverse cascade

Evolution in a dense turbulent plasma (after inflation):

- **Viscosity damping** on small scales
- **Selective decay** of non-helical magnetic field spectrum
- **Inverse cascade** of helical magnetic field spectra due to **helicity conservation**





from: **Campanelli**, PRL 98 (2007) 251302

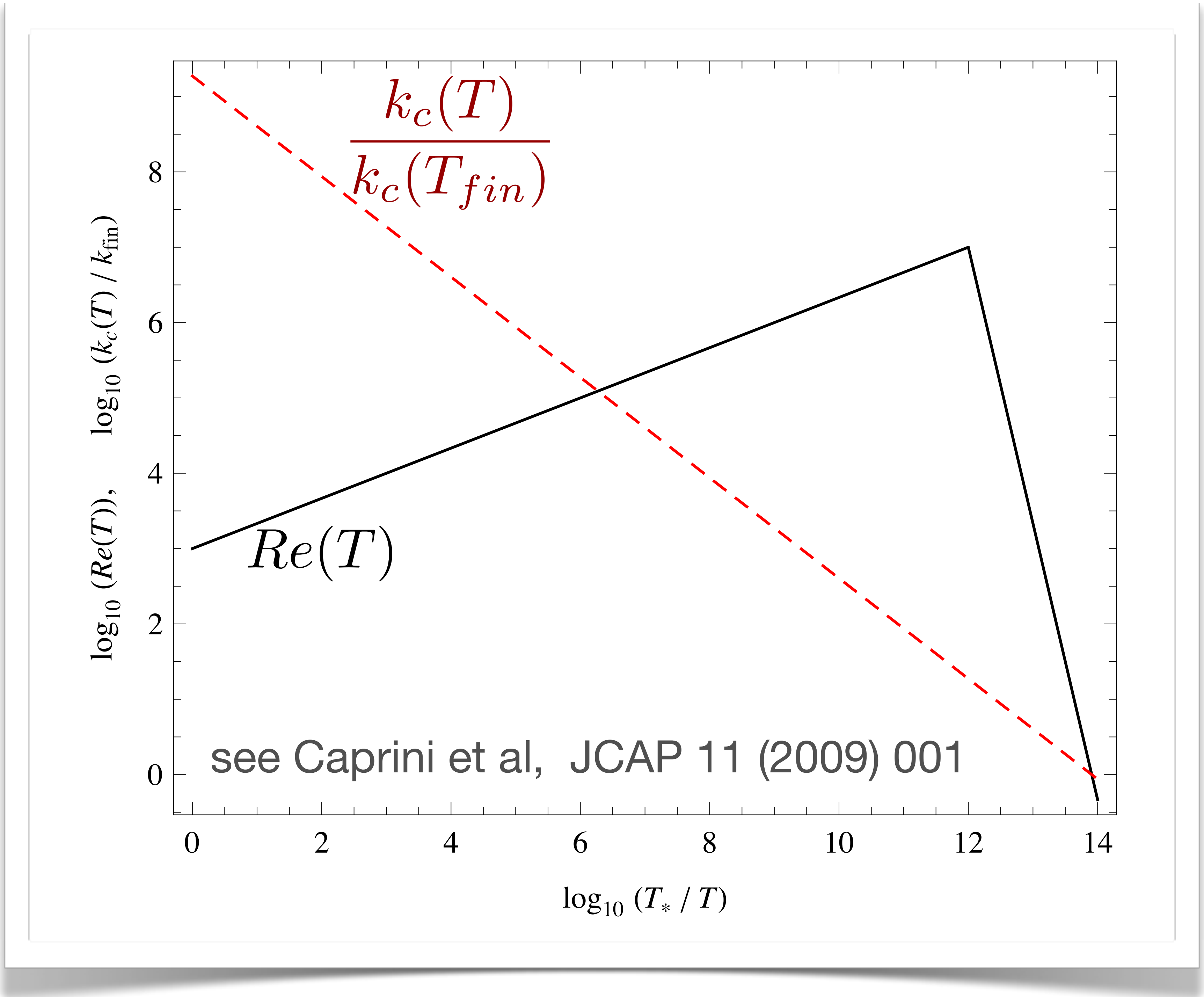
Reynolds number & correlation scale

- After inflation we deal with “ideal MHD”.
- Reynolds number characterises the turbulence

$$Re = \frac{v_L L}{\nu}$$

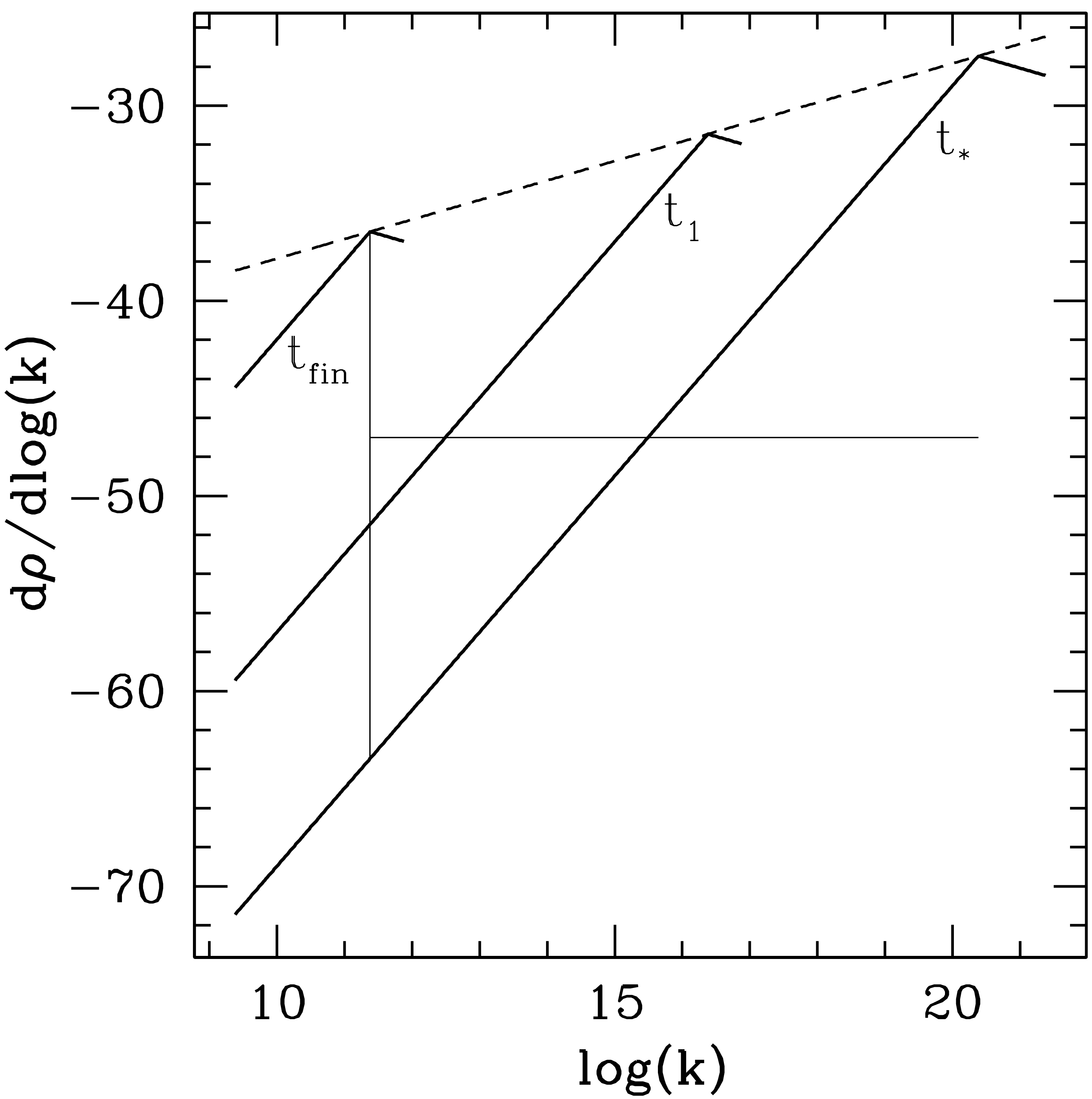
- Correlation scale evolves as

$$k_c \propto \left(\frac{T_\star}{T}\right)^{2/3}$$



as found numerically by Campanelli, PRL 98 (2007) 251302.

Magnetic power spectrum after turbulence



- On large scales the shape of the spectrum stays unchanged.
- The amplitude increases because the correlation scale grows and the total helicity is conserved

$$\mathfrak{H} \propto \rho_B / k_c$$

Final magnetic field strength

- In terms of the final correlation scale:

$$\widetilde{B}(k) \simeq 3 \times 10^{-19}\,\mathrm{Gauss}\,\mathcal{S}(f_N)\left(\frac{k}{k_{\mathrm{fin}}}\right)^2\left(\frac{T_*}{10^{14}\,\mathrm{GeV}}\right)^{19/11}$$

- And at scales of 0.1 Mpc:

$$\widetilde{B}(k=10/\mathrm{Mpc}) \simeq 3 \times 10^{-41}\,\mathrm{Gauss}\,\mathcal{S}(f_N)\left(\frac{T_*}{10^{14}\,\mathrm{GeV}}\right)^{9/11}$$

- With the bound from **backreaction**, we get at most

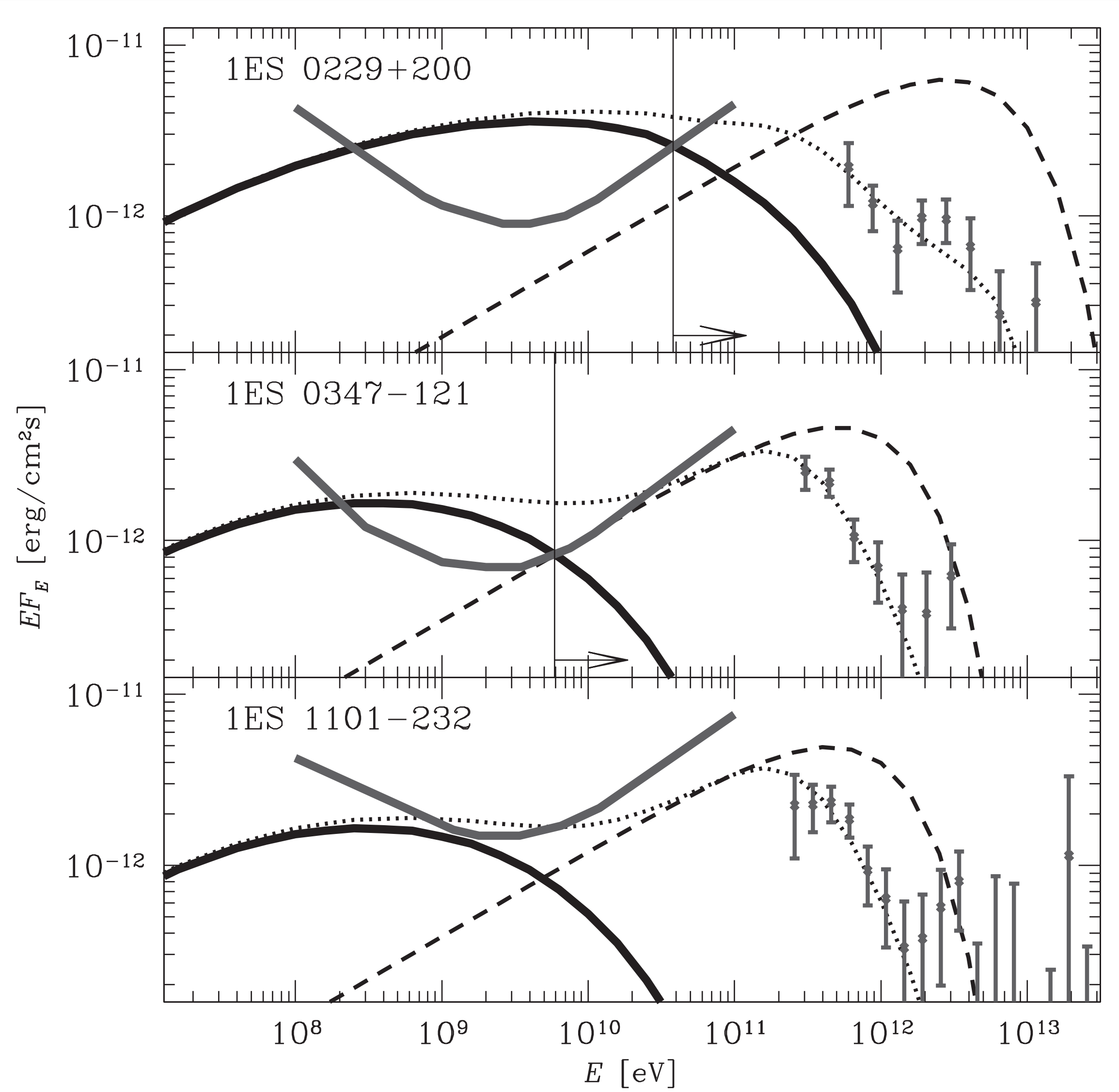
$$\widetilde{B}(k=10/\mathrm{Mpc}) \leq 10^{-32}\,\mathrm{Gauss}\left(\frac{10^{14}\,\mathrm{GeV}}{T_*}\right)^{13/11}$$

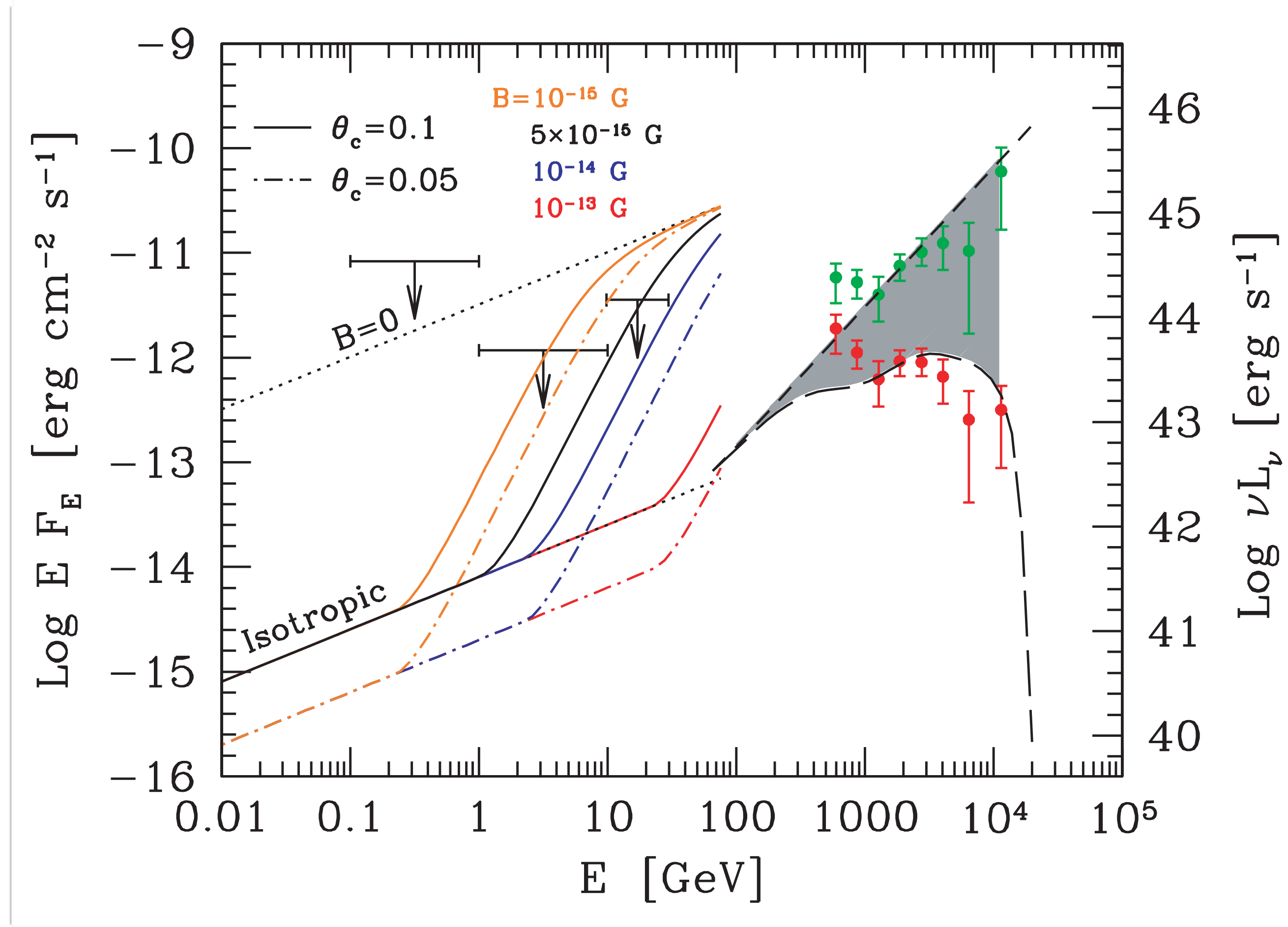
- NOT ENOUGH to seed subsequent astrophysical amplification.

Conclusions

- Helical magnetic field generation is typical for inflation scenarios with pseudo-scalars -> Neil and Lorenzo's talks
- Typically, helical magnetic field are only produced with a **blue spectrum ~ k**. (CMB data seems to prefer white or red spectra slightly.)
- Difficult to produce large amplitude without **backreaction**, large **non-Gaussianity,** and other complications.
- Although the inverse cascade can move power from small to large scales, this is **insufficient** to account for the observed fields on large scales.

Observations: lower limit from gamma-rays





from: **Neronov & Vovk**, Science 328 (2010) 73

from: **Tavecchio et al.**, MNRAS 406 (2010) L70